

# **Classifying Estimated Corresponding Points by Delaunay Triangulation**

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- ❑ Stereo Vision
- ❑ Applications
- ❑ Correspondence Problem
- ❑ Objectives
- ❑ State of the Art
- ❑ Proposed Approach
  - Feature Points
  - Initial Correspondences
  - Delaunay Triangulation
  - Constrains
  - Classification
  - Verification
- ❑ Results
- ❑ Conclusions

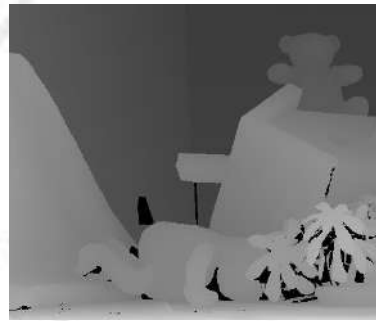
Stereo Images



Left

Right

Correspondence  
Algorithm

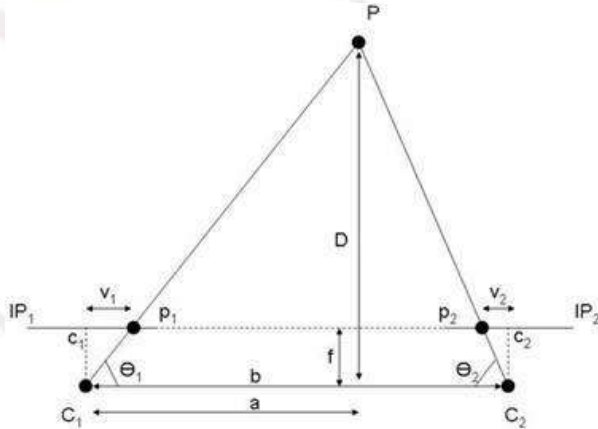


Disparity Map

3D Model



Reconstruction  
Algorithm





**Automotive**



**Entertainment**



**Industry**



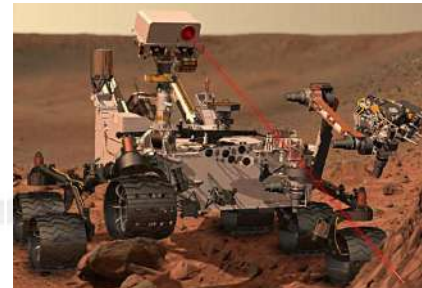
**Medical**



**Military**



**Robotics**



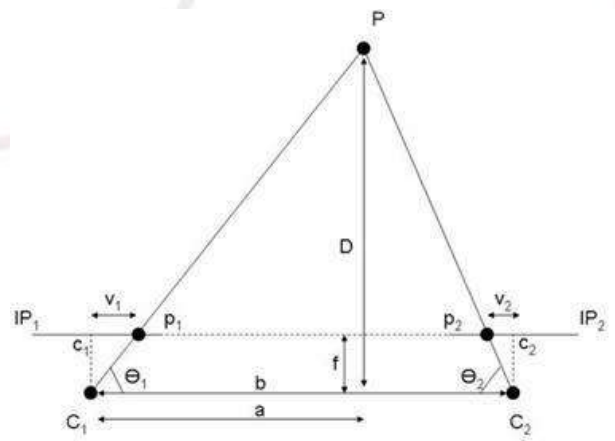
**Space**



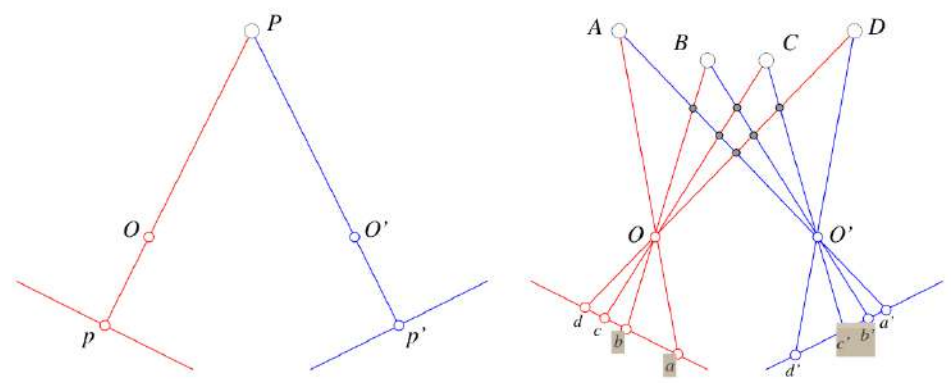
**Training**

<http://news.investors.com/article/618922/201207201352/autonomous-cars-google-self-driving-prius.htm?p=full>  
<http://maoh29.blogspot.com/2010/11/y-que-de-la-realidad-virtual-para.html>  
[http://opencv.willowgarage.com/wiki/GSOC\\_OpenCV2011](http://opencv.willowgarage.com/wiki/GSOC_OpenCV2011)  
[http://ricardogupi.blogspot.com/2010\\_12\\_01\\_archive.html](http://ricardogupi.blogspot.com/2010_12_01_archive.html)  
<http://mars.cs.umn.edu/projects/current/PerformanceCLATT/PerformanceCLATT.html>  
<http://mech4eng.blogspot.com/2011/09/robotics.html>  
<http://www.csmonitor.com/Science/2012/0731/Five-essential-facts-about-NASA-s-Mars-Curiosity-rover-video>  
<http://www.hizook.com/blog/2009/08/17/immersive-man-machine-interface-teleoperation-rollin-justin-humanoid-robot>

# Correspondence Problem



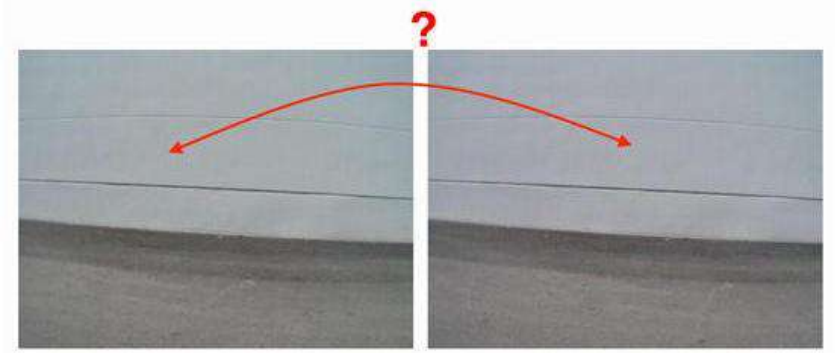
**Inverse problem**



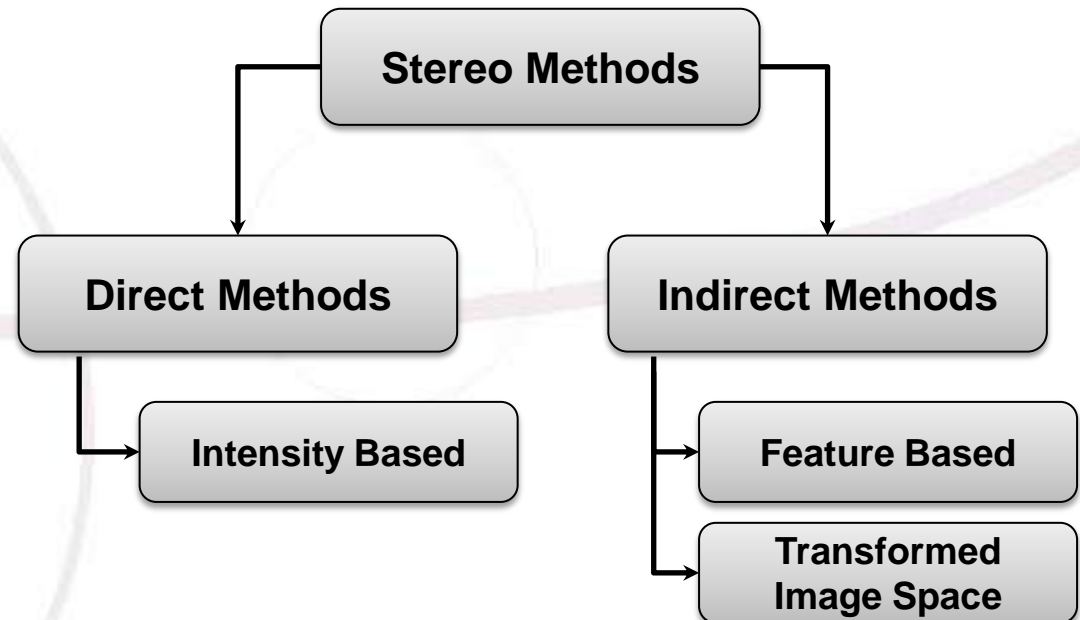
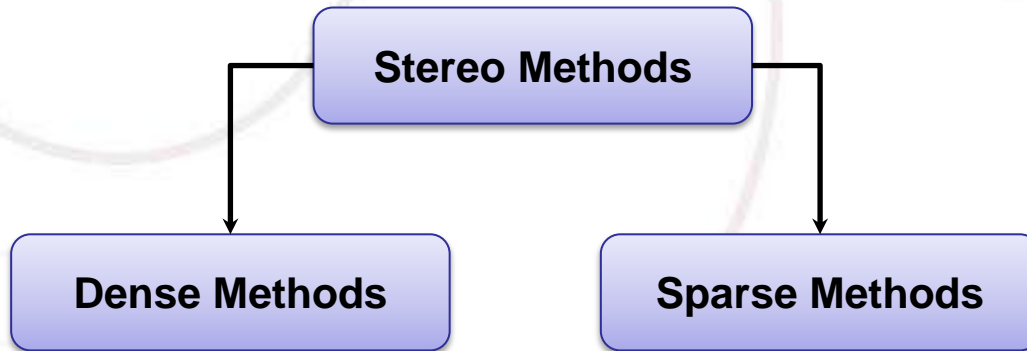
**false-target problem**

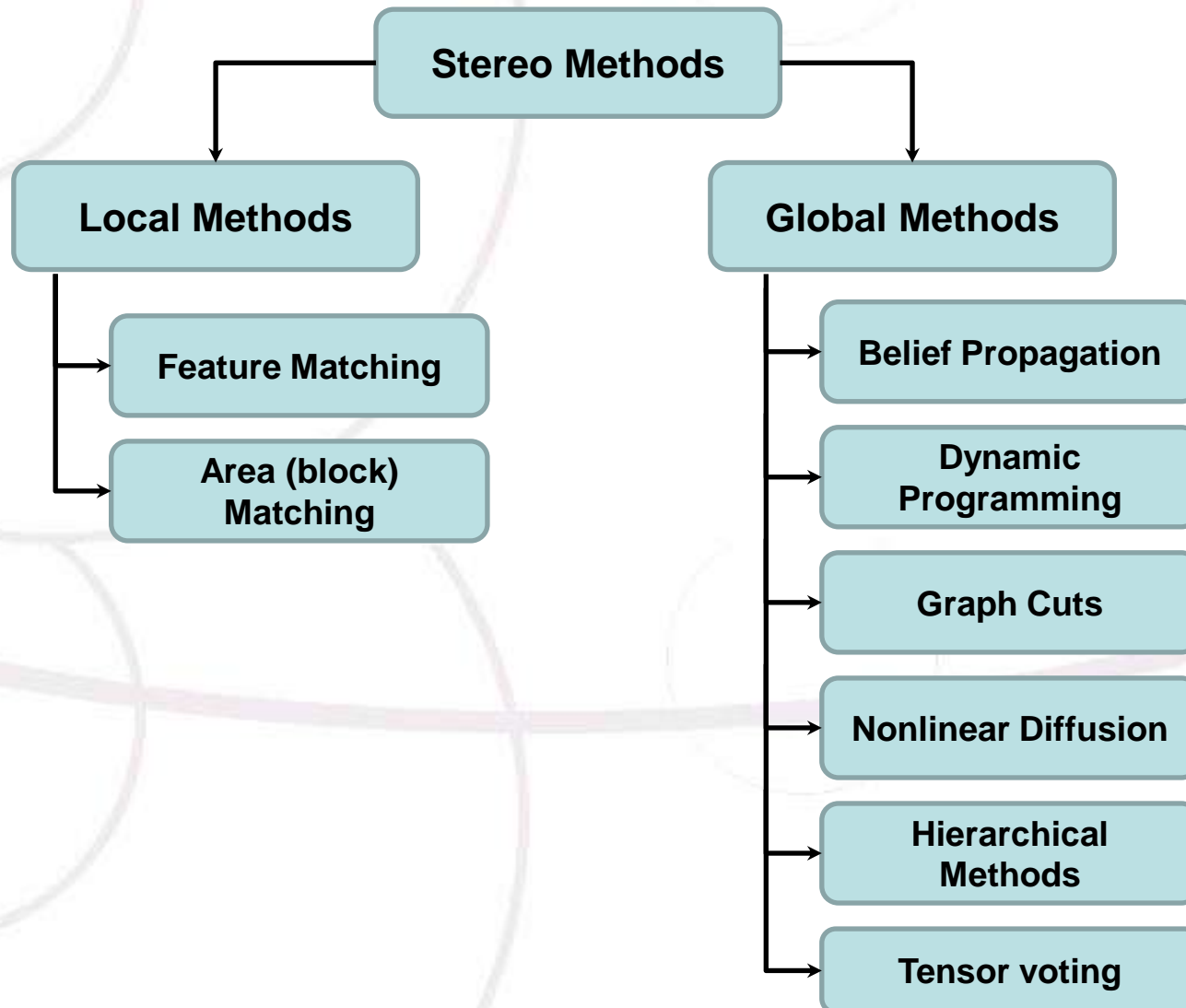


**Occlusions**



**Textureless regions**





## State of the Art (III)

- Establishing dense correspondence maps
- Allows estimating large displacements and subsequently taking into account motion/disparity discontinuities
- Computationally efficient

**Estimation of large-amplitude motion and disparity fields: Application to intermediate view reconstruction**

**Efficient Large-Scale Stereo Matching**

- Generative probabilistic model
- Bayesian approach
- 2D mesh via Delaunay triangulation

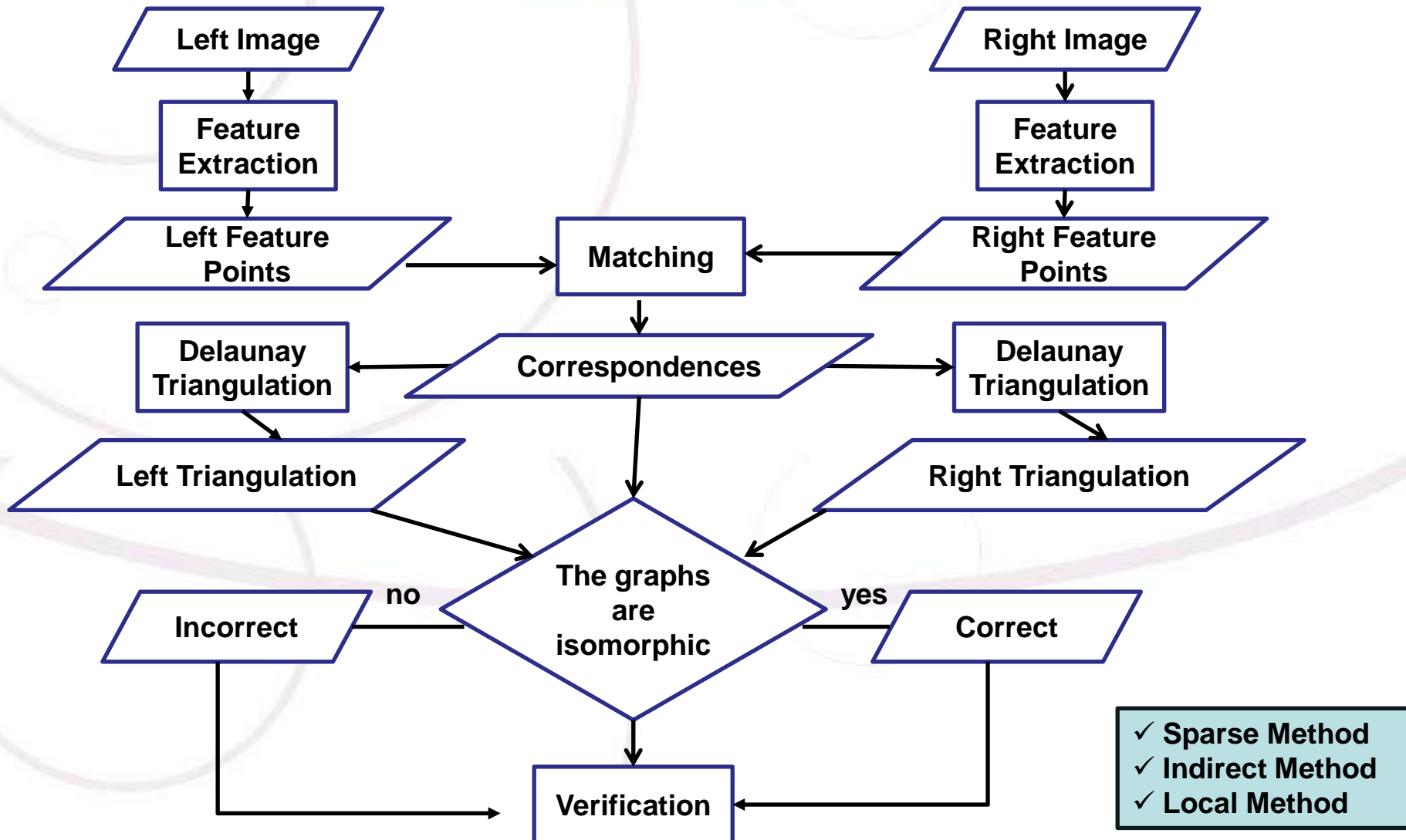
- Image pairs are triangulated
- Triangles are classified into matched and unmatched triangles
- A dense disparity map of the image is obtained

**A Dense Stereo Matching Algorithm Based on Triangulation**

**Effective Corner Matching Based on Delaunay Triangulation**

- Input images are partitioned into small and localised regions
- Point correspondences were established by planar homographies





## Scale Invariant Feature Transform (SIFT)



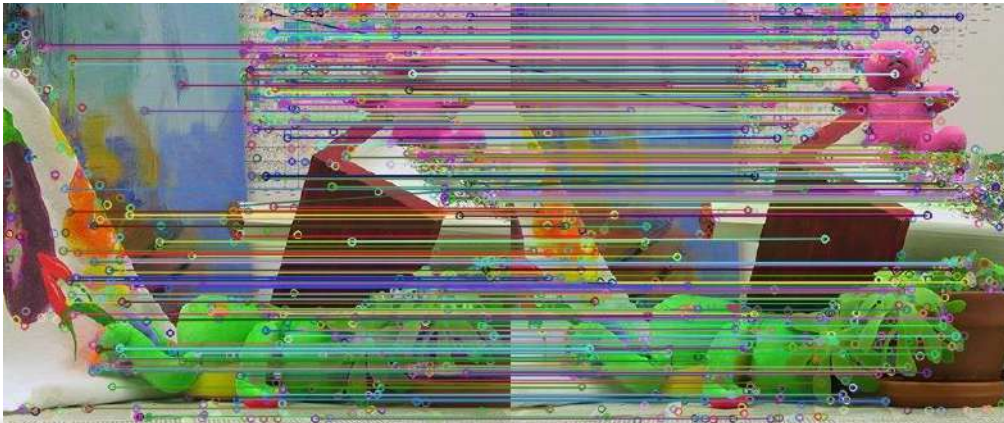
- Invariant to image scaling and rotation
- Partially invariant to change in illumination and 3D camera viewpoint
- Well localized in both the spatial and the frequency domains
- Reduce the probability of disruption by occlusion, clutter, or noise

## Features from Accelerated Segment Test (FAST)

- High quality corner detector
- Implemented using machine learning
- Several orders faster than other corner detectors
- High levels of repeatability under large aspect changes and for different kind of features



## Scale Invariant Feature Transform (SIFT)



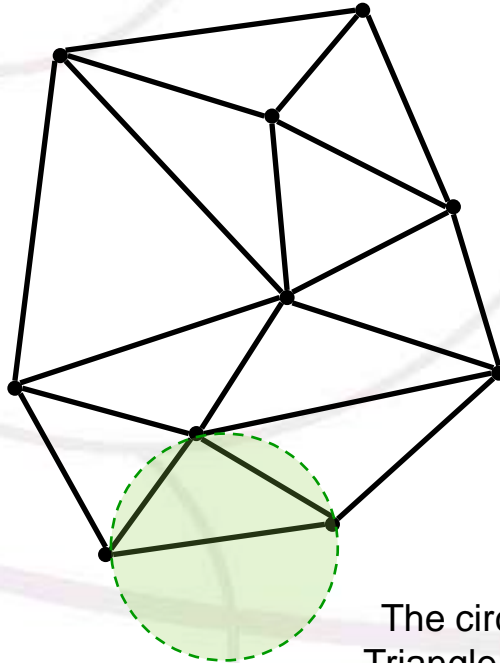
- The matching criteria for the best candidate match for each keypoint is found by identifying its nearest neighbour
- The nearest neighbour is defined as the keypoint with the minimum Euclidean distance for the invariant descriptor vector

## Features from Accelerated Segment Test (FAST)

- Block matching strategy using SDD (Sum of squared differences) of the corners descriptor
- The number of matches obtained for each stereo par is variable, but it is around 150 and 300



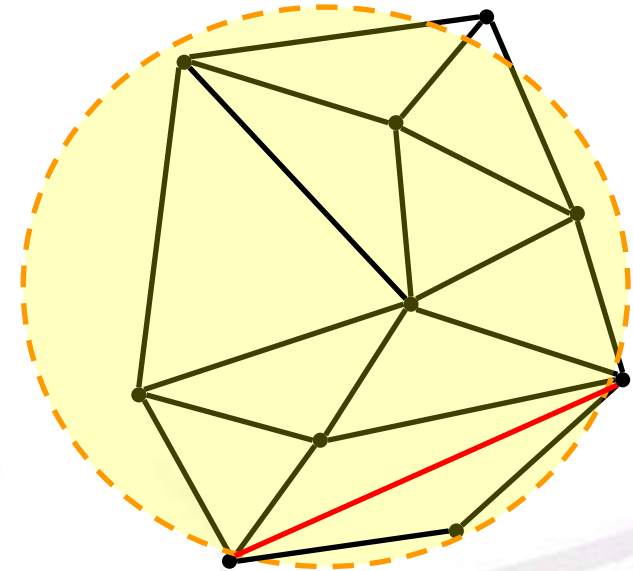
It is unique



The circumcircle of any  
Triangle does not contain  
a point of P in its interior

**Delaunay Triangulation**

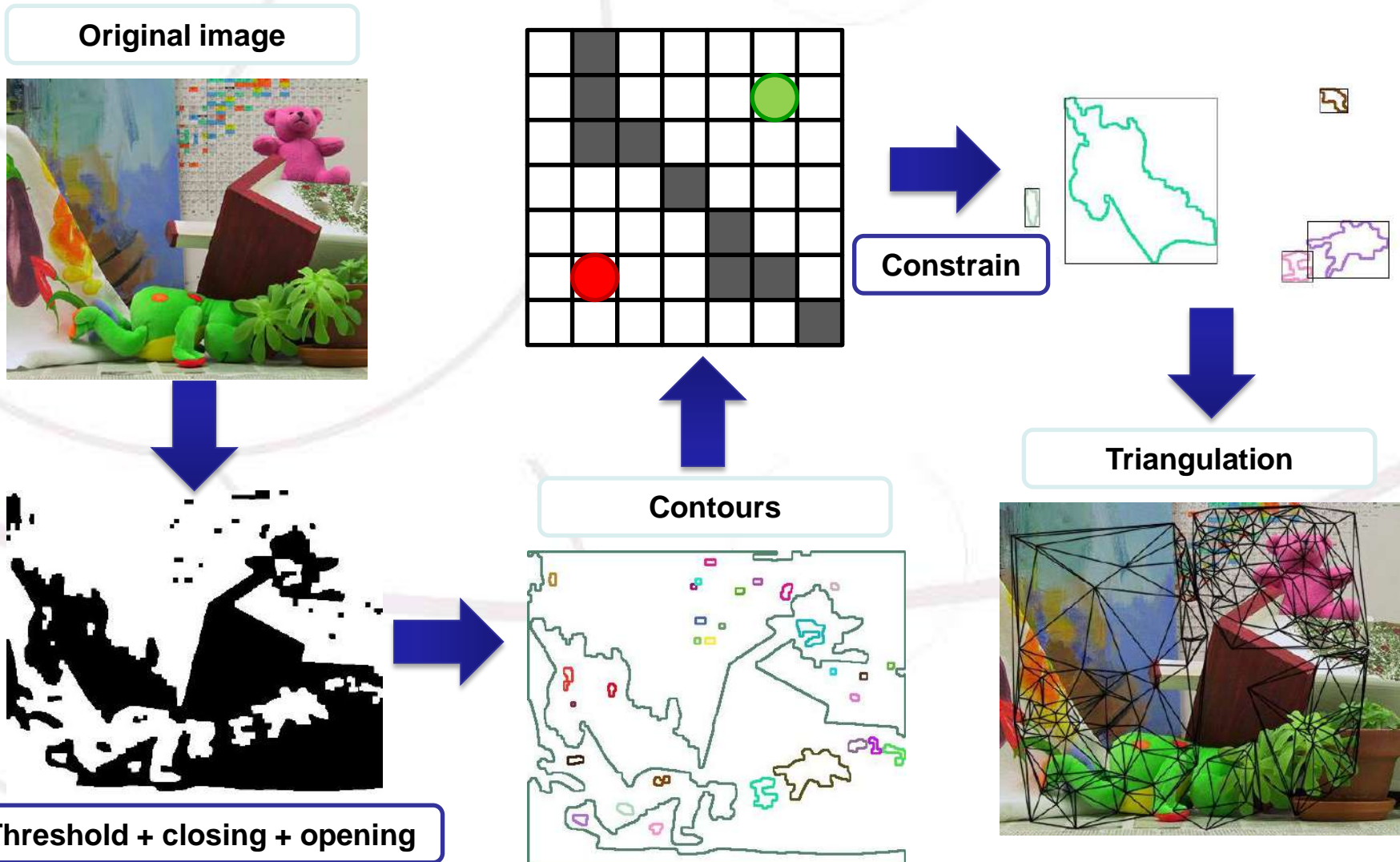
Maximizes the  
minimum angle  
over all  
triangulations of  
P



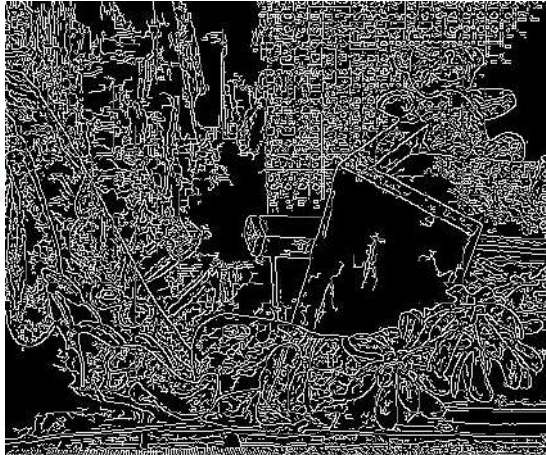
It is not completely  
Delaunay

**Constrained Delaunay Triangulation**

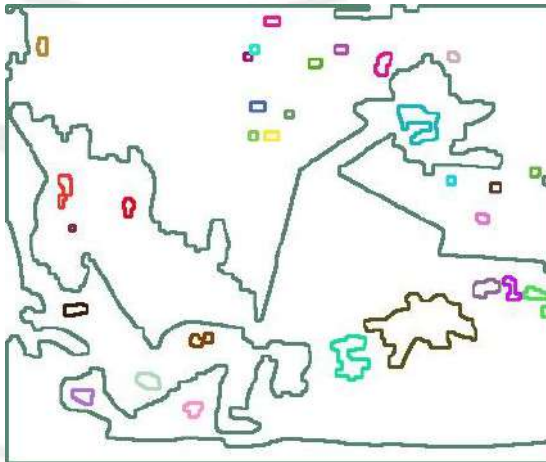
# Triangles Constrain



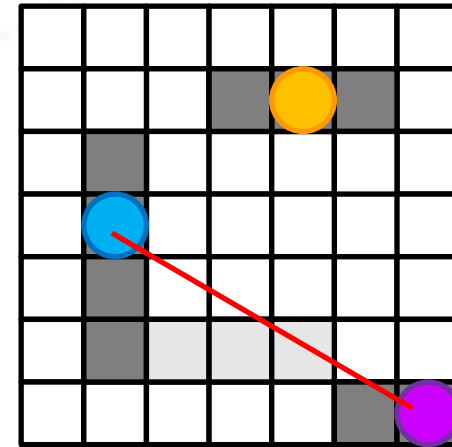
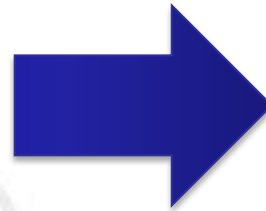
# Edges Constrain



Edges

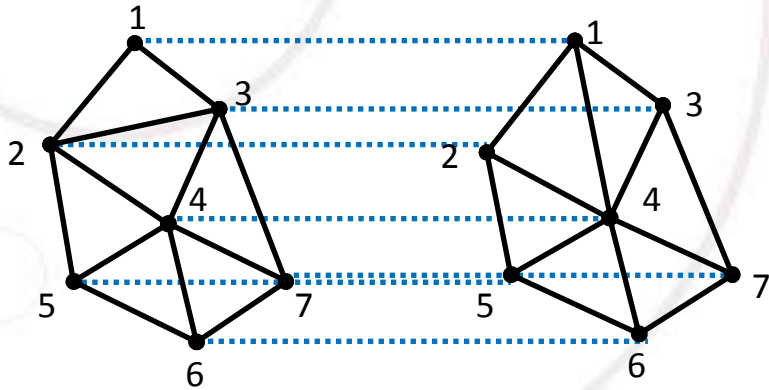


Contours

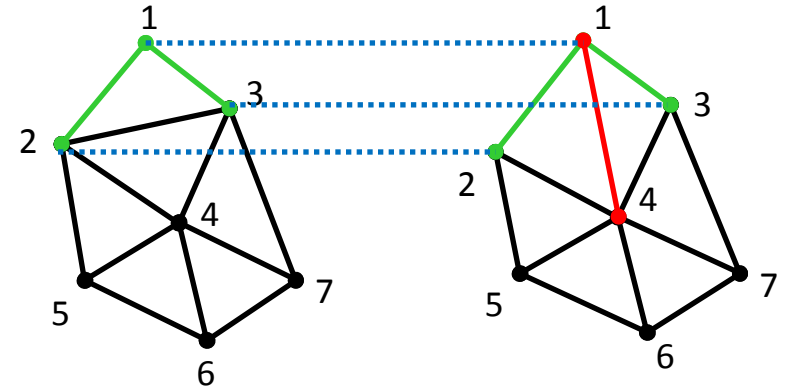


Constrain

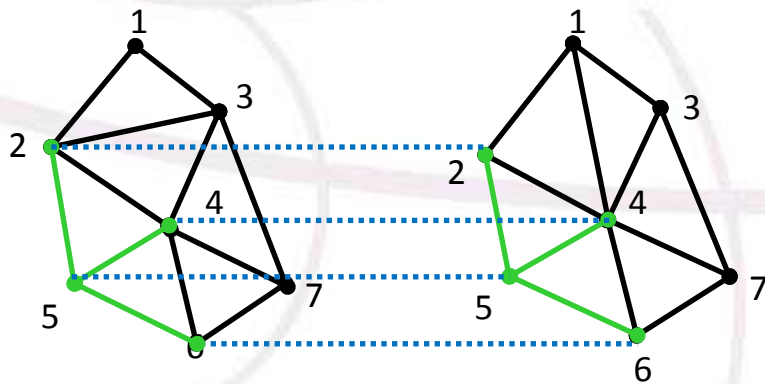




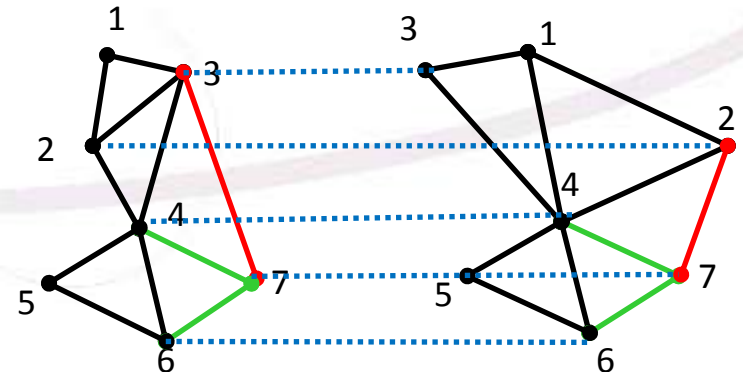
Triangulation of a set of corresponding points



Classification criteria using the vertex number 1



Classification criteria using the vertex number 5



Classification criteria using the vertex number 7

Given a set of corresponding points, they are mapped into an undirected graph and corresponding points are classified as “correctly estimated” if and only if their graphs are isomorphic

*Let  $G$  and  $G'$  be a set of corresponding points, from the right and the left images respectively. A Delaunay triangulation produces a set of vertices and edges  $(V, A)$  and  $(V', A')$  respectively. A bijective function  $f: V \rightarrow V'$  is a graph isomorphism if:*

$$w, v, z \in A \leftrightarrow \phi(v), \phi(w), \phi(z) \in A'$$

That is, if  $f$  preserves the adjacency between vertices,

The bijective function  $f$  is represented by the initial map of correspondences and the set of adjacent vertices.



**if** left.numberOfVertex > 3 **then**

**if** right.vertex *equal* (left.vertex - 1) **or** right.vertex *equal* left.vertex **then**

**mark as** correct estimated

**else**

**mark as** incorrect estimated

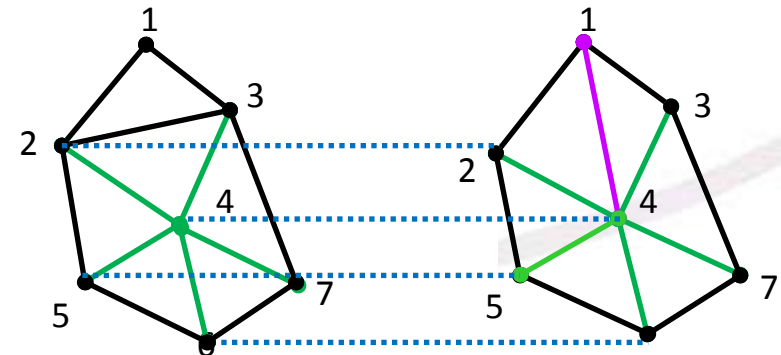
**else**

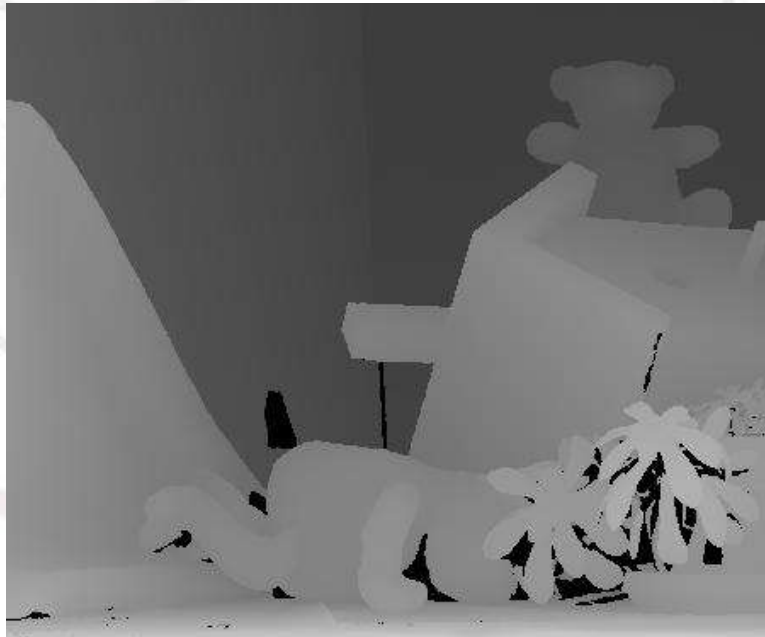
**if** right.vertex *equal* left.vertex **then**

**mark as** correct estimated

**else**

**mark as** incorrect estimated





Ground Truth Image

$$error_{i,j} = |GT_{i,j} - \Delta x_{i,j}|$$

$GT_{i,j}$ : ground-truth disparity value at (i,j)

$\Delta x_{i,j}$ : estimated disparity value at (i,j)

$error > 1$  : “bad match”

$error < 1$  : “good match”



Art



Books



Cones



Dolls



Laundry



Moebius



Reindeer



Teddy

<http://vision.middlebury.edu/stereo/data/scenes2003/>  
<http://vision.middlebury.edu/stereo/data/scenes2005/>

## ***Sensitivity***

$$\text{sensitivity} = \frac{tp}{tp + fn}$$

*Probability that is classified as  
“bad match”,  
Given that is really a “bad  
match”*

## ***Specificity***

$$\text{specificity} = \frac{tn}{tn + fp}$$

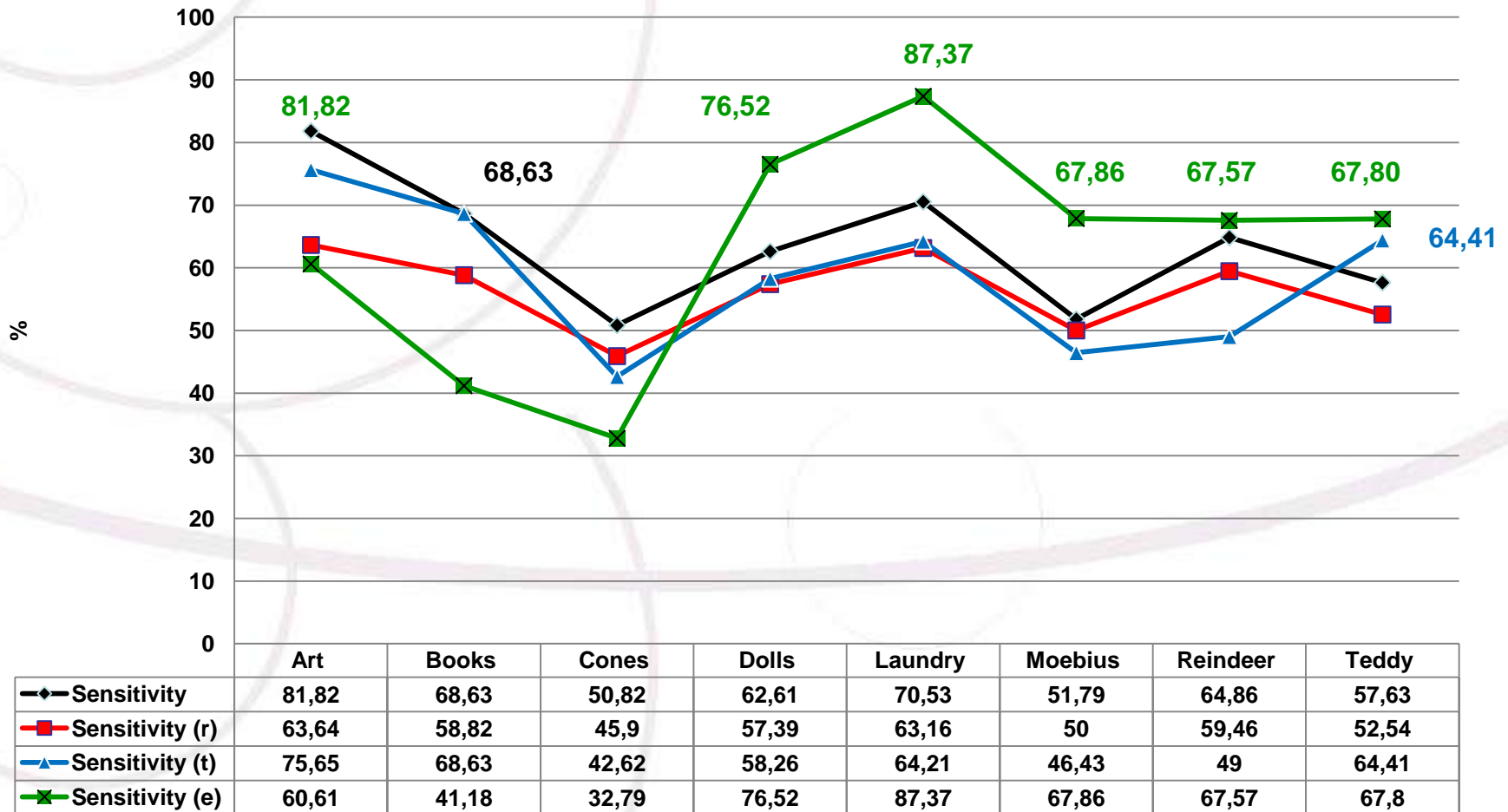
*Probability that is classified as  
“good match”,  
Given that is really a “good  
match”*

tp: true positives  
fp: false positives  
tn: true negatives  
fn: false negatives

## SIFT Algorithm Performance (Initial Matching vs Ground-Truth)

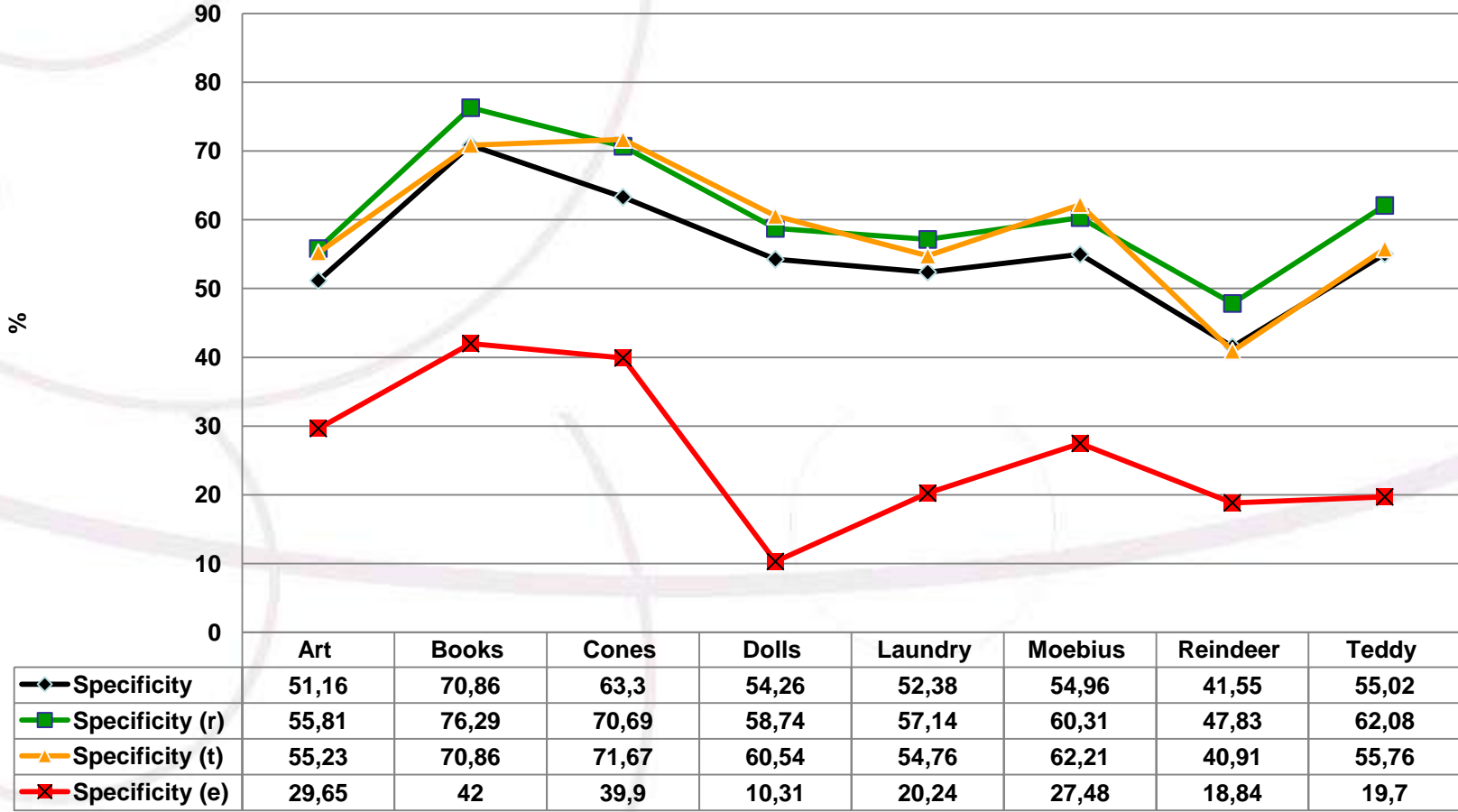
Dataset	Points Left	Points Right	Matches	Bad Matches	% Bad Matches
Art	1167	1089	205	33	16,10
Books	823	904	401	51	12,72
Cones	1163	1147	467	61	13,06
Dolls	1462	1406	561	115	20,50
Laundry	1123	1124	263	95	<b>36,12</b>
Moebius	867	920	318	56	17,61
Reindeer	592	553	244	37	15,16
Teddy	844	881	328	133	<b>40,55</b>

## Sensitivity for SIFT



(r) = relaxed condition  
(t) = triangles constrain  
(e) = edges constrain

## Specificity for SIFT



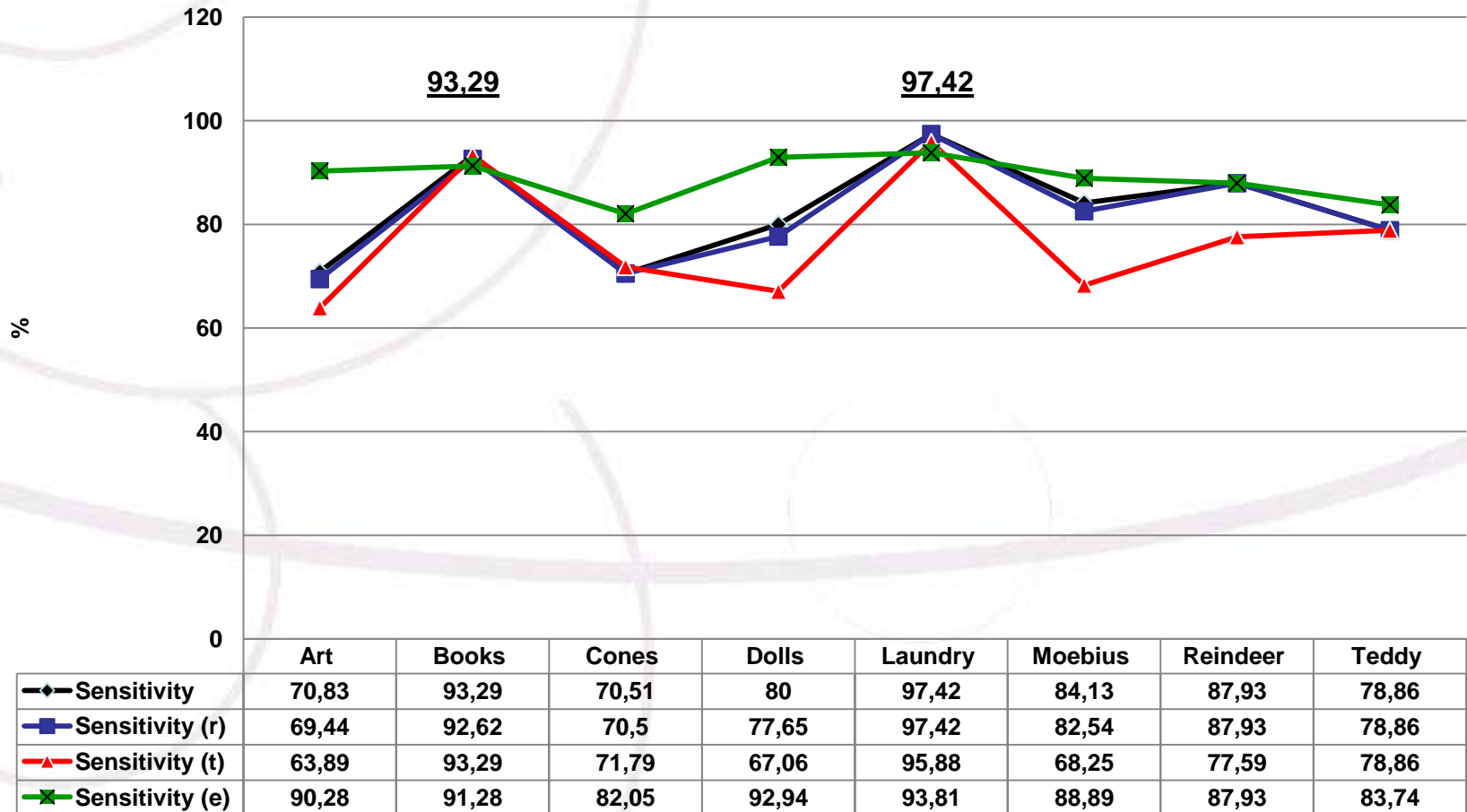
(r) = relaxed condition  
 (t) = triangles constrain  
 (e) = edges constrain

## Corners Algorithm Performance (Initial Matching vs Ground-Truth)

Dataset	Points Left	Points Right	Matches	Bad Matches	% Bad Matches
Art	985	986	164	72	43,90
Books	958	955	303	149	<b>49,17</b>
Cones	947	946	268	78	29,10
Dolls	950	948	280	85	30,36
Laundry	960	966	293	194	<b>66,21</b>
Moebius	963	970	213	63	29,58
Reindeer	948	955	201	58	28,86
Teddy	950	947	316	123	38,92

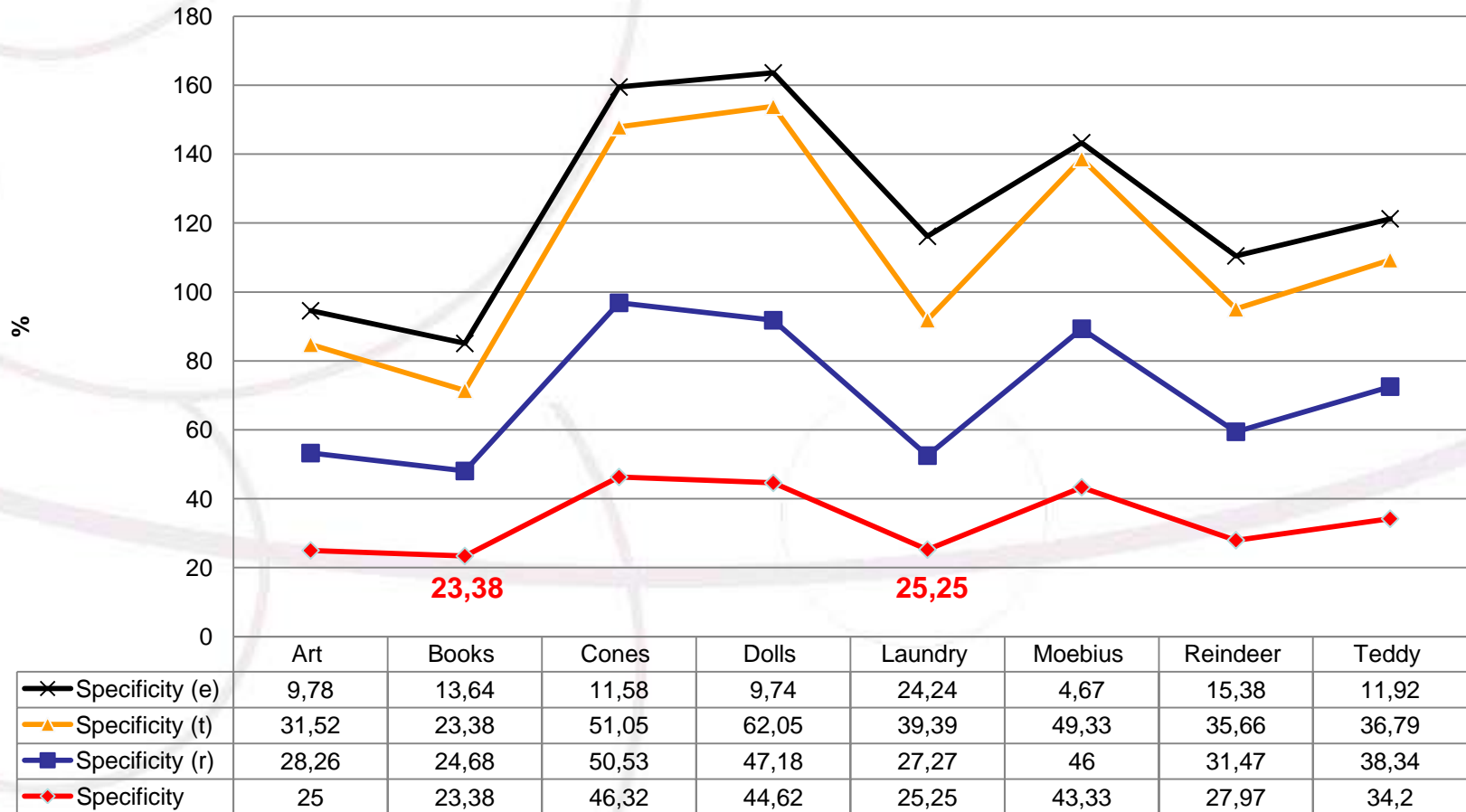


## Sensitivity for Corners



(r) = relaxed condition  
(t) = triangles constrain  
(e) = edges constrain

## Specificity for Corners



(r) = relaxed condition  
(t) = triangles constrain  
(e) = edges constrain

- ❑ The use of all the recognized keypoints for the triangulation, **adds noise** to it
- ❑ The classification algorithm **presents better performance for the corners than for the SIFT keypoints**, in terms of **sensitivity**, which is for our analysis the most representative measurement of quality. On the other hand, the **specificity is higher for the SIFT algorithm**
- ❑ The classification algorithm exhibit the **best performance** in the **corners feature points**, where the percentage of initial bad matches is bigger than approximately **50%**. It means, that the algorithm presents **high sensitivity** (between 93% and 97%) **when the initial matching is bad**
- ❑ The use of **constrains** contribute to the **increment of specificity**. However, it also **decrements the sensitivity**
- ❑ The **relaxed evaluation condition** presented **worked better** for the **corners features**, because of the locations of the points